I – Special Section on Nonsmooth Analysis, Control and Optimization
Guest Editorial

The need to provide satisfactory answers to several questions posed by advanced applications in Control has been one of the strongest motivations to tackle mathematical problems where differentiability of the involved functions is no longer guaranteed. Thus, a completely different approach was developed, where set-valued replacements for the usual notion of derivative are introduced. The term Nonsmooth Analysis refers to the body of theory which develops differential calculus for functions which are not differentiable in the usual sense, and for sets which are not classical smooth manifolds.

Nonsmooth analysis has played an increasingly important role in several areas of application, notably in control theory, optimization, calculus of variations, differential equations, aeronautics, economics and mechanics. In the case of control theory, the need for nonsmooth analysis first came to light in connection with finding proofs of necessary conditions for optimal control, notably in connection with the Pontryagin maximum principle. This necessity holds even for problems which are expressed entirely in terms of smooth data. Subsequently, it became clear that problems with intrinsically nonsmooth data arise naturally in a variety of control settings. Nonsmooth analysis enters the picture as soon as we consider problems which are nonlinear, whether for deriving or expressing necessary conditions, in applying sufficient conditions, or in studying the sensitivity of the problem. A considerable body of theory on optimal control has now been built upon of nonsmooth analysis results. Recently, nonsmooth analysis has played a crucial role in the study of stability of control systems. It is now clear that in the analysis of truly nonlinear control systems, the consideration of nonsmooth Lyapunov functions and discontinuous feedbacks becomes unavoidable.

In this special section, we reveal developments of nonsmooth analysis and its contributions to control and optimization. Most papers that constitute the special section were based on works presented at the Workshop in Control, Nonsmooth Analysis and Optimization, that was held in Porto, Portugal, in May 2009. The workshop celebrated the 60th birthday of Francis H. Clarke and Richard B. Vinter, two of the most influential researchers in nonsmooth analysis and its applications to optimization and control theory. Each paper in this special section is briefly described below.

The paper by Zvi Artstein, *Pontryagin Maximum Principle Revisited with Feedbacks*, proposes a systematic approach to extract information from the Maximum Principle by considering appropriate feedback controls. The author concludes with a discussion on the maximum principle.

The paper by Martino Bardi and Annalisa Cesaroni, *Optimal Control with Random Parameters: A Multiscale Approach*, considers a stochastic optimal control problem with “fast” and “slow” dynamics. It establishes, under appropriate assumptions, that when the velocity parameter of the fast dynamics goes to infinity, the optimal solution converges to a solution that can be obtained from viscosity solutions of the Hamilton-Jacobi-Bellman equation corresponding to an “effective” (averaged) Hamiltonian function.

The paper by Zbigniew Bartosiewicz, Natália Martins and Delfim F. M. Torres, *The Second Euler-Lagrange Equation of Variational Calculus on Time Scales*, gives an extension of the DuBois-Reymond equation of the calculus of variations (so called “second Euler-Lagrange equation”) to variational problems on time scales, that is, for problems for which the time is not necessarily continuous but an arbitrarily nonempty closed set of the real numbers.

The paper by Bahne Christiansen, Helmut Maurer and Oliver Zirn, *Optimal Control of Servo Actuators with Flexible Load and Coulombic Friction*, addresses the improvement of the positioning and process duration of an electro-dynamical servo drive system (the voice-coil motor) by means of optimal control techniques. The results of the paper...
present a practical approach for solving nonsmooth optimal control problems. Due to Coulomb friction, state-dependent discontinuities may occur in the dynamics and in that case the problem is a nonsmooth optimal control problem.

The paper by Vladimir Dykhta and Olga Samsonyuk, *Some Applications of Hamilton-Jacobi Inequalities for Classical and Impulsive Optimal Control Problems*, concerns the development of the so-called non-convex duality for impulsive control systems modeled by measure-driven differential equations.

Finally, we would like to thank all the authors who contributed to this special section. We trust that the results here presented constitute a significant and stimulating scientific opportunity for the control community, illustrate new results and applications of nonsmooth analysis in control and highlight promising research directions arising from applications of control theory and optimization.

*Guest Editors*

MARIA DO ROSÁRIO DE PINHO
FERNANDO A. C. C. FONTES
DELFIM F. M. TORRES
University of Aveiro Portugal